

DBFT: Efficient Byzantine Consensus with a Weak Coordinator and its Application to Consortium Blockchains *

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Abstract

This paper introduces a deterministic Byzantine consensus algorithm that relies on a new *weak coordinator*. As opposed to previous algorithms that cannot terminate in the presence of a faulty or slow coordinator, our algorithm can terminate even when its coordinator is faulty, hence the name weak coordinator. The key idea is to allow processes to complete asynchronous rounds as soon as they receive a threshold of messages, instead of having to wait for a message from a coordinator that may be slow.

The resulting algorithm assumes partial synchrony, is resilience optimal, time optimal and does not need signatures. Our presentation

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is didactic: we first present a simple safe binary Byzantine consensus algorithm, modify it to ensure termination, and finally present an optimized reduction from multivalued consensus to binary consensus that may terminate in 4 message delays.

To evaluate our algorithm, we deployed it on 100 machines distributed in 5 datacenters across different continents and compared its performance against the randomized solution from Mostéfaoui, Moumen and Raynal [PODC'14] that terminates in $O(1)$ rounds in expectation. Our algorithm always outperforms the latter even in the presence of Byzantine behaviors. Our algorithm has a subsecond average latency in most of our geo-distributed experiments, even when attacked by a well-engineered coalition of Byzantine processes.

1 Introduction and Related Work

To circumvent the impossibility of solving consensus in asynchronous message-passing systems [24] where processes can be faulty or *Byzantine* [31], researchers typically use randomization [5, 6, 15] or additional synchrony assumptions.

Randomized algorithms can use per-process “local” coins or a shared “common” coin to solve consensus probabilistically among n processes despite $t < \frac{n}{3}$ Byzantine processes. When based on local coins, the existing algorithms converge in $O(n^{2.5})$ expected time [27]. A recent randomized algorithm without signature [37] solves consensus in $O(1)$ expected time under a fair scheduler. The fair scheduler assumption was later relaxed in an extended version [38] that we refer to as *Coin* in the remainder of the paper. Unfortunately, implementing a common coin increases the message complexity of the consensus algorithm.

To avoid the need of a common coin and solve the consensus problem *deterministically*, researchers have assumed partial or eventual synchrony [22]. Interestingly, these solutions typically require a unique *coordinator* process, sometimes called a leader, to be non-faulty [3, 8, 16, 21, 22, 28, 32, 34]. The advantage is that if the coordinator is non-faulty and if the messages are delivered in a timely manner in an asynchronous round, then the coordinator broadcasts its proposal to all processes and this value is decided after a constant number of message delays. The drawback is that a faulty coordinator can dramatically impact the algorithm performance by leveraging the power it has in a round and imposing its value to all. Non-faulty processes thus have no other choices but to decide nothing in this round.

In this paper, we present a *weak coordinator* alternative that does not suffer from this drawback. It allows us to introduce a new deterministic

Byzantine consensus algorithm that is time optimal, resilience optimal and does not need signatures. As opposed to a classic (strong) coordinator, the weak coordinator does not impose its value. On the one hand, this allows non-faulty processes to decide a value quickly without the help of the coordinator. On the other hand, the coordinator helps the algorithm terminating if non-faulty processes know that they proposed distinct values that might all be decided. Furthermore, having a weak coordinator allows rounds to be executed optimistically without waiting for a specific message. This differs from classic BFT algorithms [16] that have to wait for a particular message from their coordinator and sometimes have to recover from a slow network or faulty coordinator.

To mitigate the problem of a slow or Byzantine coordinator, other approaches were previously explored. Some protocols progressively reduce the time allocated to a coordinator to solve consecutive consensus instances in order to force the change of a slow coordinator [4, 20]. While this still requires a (strong) coordinator in each round, it favors the fastest coordinator in successive rounds. An exponential information gathering tree was used to terminate in $t + 3$ rounds without a coordinator [9]. Other solutions [22, 45] require at least $O(t)$ rounds. By contrast our weak coordinator only helps agreement by suggesting a value while still allowing termination in a constant number of message delays and thus differs from the classic coordinator [17, 22] or the eventual leader that cannot be implemented in $\mathcal{BAMP}_{n,t}[t < n/3]$.

Application to consortium blockchains. To motivate our algorithm, we study its applicability to the recent context of *blockchains* [40]. Blockchains originally aimed at tracking ownerships of digital assets where any Internet user could solve a cryptopuzzle before proposing, for consensus, a block of asset transactions. The *consortium blockchains* [12] became promising at reducing the amount of resources consumed by avoiding to resolve the cryptopuzzle but restricting the set of proposers to n known processes.

These consortium blockchains seem similar to replicated state machines [30, 44] where a sequence of commands must be decided by multiple processes. Some blockchains already use Byzantine fault tolerant consensus, for example, Hyperledger [2] uses a consensus based on a classic coordinator [8], Honeybadger [36] uses a randomized algorithm [37] and the Red Belly Blockchain [25] uses a preliminary version of the algorithm we introduce here [19]. A slight difference with state machine replication is that the block at index x of a blockchain must embed the hash of the block decided at instance number $(x - 1)$. This relation between instances is interesting as it entails a natu-

ral mechanism during a consensus instance for discarding fake proposals or, instead, extracting a *valid* value out of various proposals.

We thus propose a variant of the consensus problem that allows us to extend common definitions of Byzantine consensus, that either assume that no value proposed only by Byzantine processes can be decided [18, 38, 39], or that any value (i.e., possibly proposed by a Byzantine process) can be decided [22, 26, 33, 34, 43]. Interestingly, the validity property we propose allows a decided value to combine multiple proposals but is less strict than interactive consistency [42] or vector consensus [41]: for example, it does not require the decided value to combine at least $t + 1$ values proposed by correct processes.

Geo-distributed experimentation with Byzantine coalitions. To validate our expectations experimentally, we deployed our consensus algorithm on 100 Amazon VMs located in 5 datacenters on different continents. We also implemented “Coin” the recent randomized algorithm from Moustéoui et al. [37] used in the HoneyBadger blockchain [36] and demonstrated that under all our workloads, our algorithm outperforms “Coin” that is known to terminate in $O(1)$ round in expectation. This is due to both the overhead of the coin implementation that slows down every round and the risks of being unlucky at tossing the coin by increasing the number of rounds needed to decide.

As Byzantine behaviors are known to affect drastically performance of (strong) coordinator-based consensus [4, 20], we also implemented 4 different Byzantine attacks: Byz1 where Byzantine processes send a bit b where the protocol specification expects them to send $\neg b$; Byz2 where Byzantine processes are mute; Byz3 where Byzantine processes send a combination of random and flipped values and Byz4 where Byzantine processes form a coalition to limit the progress of non-faulty nodes from one round to another by exploiting a Byzantine coordinator and sending messages without waiting. Interestingly, the latency exceeds slightly the second only under the Byz3 attacks.

Finally, we combine our consensus algorithm with an optimized variant of the reduction of multivalued to binary consensus of Ben-Or et al. [7] to propose a novel *Democratic Byzantine Fault Tolerant (DBFT)* consensus algorithm applicable to consortium blockchains that terminates in 4 messages delays in the good case, when all non-faulty processes propose the same value.

Roadmap. Section 2 presents the model. Section 3 presents the binary

Byzantine consensus algorithm. Section 4 presents the consensus definition and an application to the blockchain context and Section 5 concludes the paper. The proofs of safety and termination as well as experimental results are deferred to the appendix.

2 A Byzantine Computation Model

Asynchronous processes. The system is made up of a set Π of n asynchronous sequential processes, namely $\Pi = \{p_1, \dots, p_n\}$; i is called the “index” of p_i . “Asynchronous” means that each process proceeds at its own speed, which can vary with time and remains unknown to the other processes. “Sequential” means that a process executes one step at a time. This does not prevent it from executing several threads with an appropriate multiplexing. Both notations $i \in Y$ and $p_i \in Y$ are used to say that p_i belongs to the set Y .

Communication network. The processes communicate by exchanging messages through an asynchronous reliable point-to-point network. “Asynchronous” means that there is no bound on message transfer delays, but these delays are finite. “Reliable” means that the network does not lose, duplicate, modify, or create messages. “Point-to-point” means that any pair of processes is connected by a bidirectional channel. Hence, when a process receives a message, it can identify its sender. A process p_i sends a message to a process p_j by invoking the primitive “send TAG(m) to p_j ”, where TAG is the type of the message and m its content. To simplify the presentation, it is assumed that a process can send messages to itself. A process p_i receives a message by executing the primitive “receive()”. The macro-operation broadcast TAG(m) is used as a shortcut for “for each $p_i \in \Pi$ do send TAG(m) to p_j end for”.

Failure model. Up to t processes can exhibit a *Byzantine* behavior [42]. A Byzantine process is a process that behaves arbitrarily: it can crash, fail to send or receive messages, send arbitrary messages, start in an arbitrary state, perform arbitrary state transitions, etc. Moreover, Byzantine processes can collude to “pollute” the computation (e.g., by sending messages with the same content, while they should send messages with distinct content if they were non-faulty). A process that exhibits a Byzantine behavior is called *faulty*. Otherwise, it is *non-faulty*. Let us notice that, as each pair of processes is connected by a channel, no Byzantine process can impersonate another process. Byzantine processes can control the network by modifying

the order in which messages are received, but they cannot postpone forever message receptions.

Additional synchrony assumption. It is well-known that there is no consensus algorithm ensuring both safety and liveness properties in fully asynchronous message-passing systems in which even a single process may crash [24]. As the crash failure model is less severe than the Byzantine failure model, the consensus impossibility remains true if processes may commit Byzantine failures. To circumvent such an impossibility, and ensure the consensus termination property, we enrich the model with additional synchrony assumptions. It is assumed that after some finite time τ , there is an upper bound δ on message transfer and process computation delays. This eventual (or partial) synchrony assumption is denoted $\diamond Synch$.

Notations. The acronym $\mathcal{BAMP}_{n,t}[\emptyset]$ is used to denote the previous basic Byzantine Asynchronous Message-Passing computation model; \emptyset means that there is no additional assumption. The basic computation model strengthened with the additional constraint $t < n/3$ is denoted $\mathcal{BAMP}_{n,t}[t < n/3]$. The latter computation model strengthened with the eventual synchrony constraint $\diamond Synch$ is denoted $\mathcal{BAMP}_{n,t}[t < n/3, \diamond Synch]$.

3 Binary Byzantine Consensus

In this section we propose a solution to the binary consensus using a weak coordinator that requires neither signatures, nor randomization. For the sake of simplicity, we build the algorithm incrementally by first recalling the binary consensus problem, then presenting a safe binary consensus algorithm in the $\mathcal{BAMP}_{n,t}[t < n/3]$ model and finally presenting a safe and live consensus algorithm in the $\mathcal{BAMP}_{n,t}[t < n/3, \diamond Synch]$ model.

Let \mathcal{V} be the set of values that can be proposed by a process to the consensus. While \mathcal{V} can contain any number (≥ 2) of values in multivalued consensus, it contains only two values in binary consensus, e.g., $\mathcal{V} = \{0, 1\}$. Assuming that each non-faulty process proposes a value, the binary Byzantine consensus (BBC) problem is for each of them to decide on a value in such a way that the following properties are satisfied:

- BBC-Termination. Every non-faulty process eventually decides on a value.
- BBC-Agreement. No two non-faulty processes decide on different values.
- BBC-Validity. If all non-faulty processes propose the same value, no other value can be decided.

3.1 The Binary Value Broadcast Communication Abstraction

Our binary consensus algorithm relies on a binary value all-to-all communication abstraction, denoted BV-broadcast, originally introduced for randomized consensus [38], and restated in the appendix.

In a BV-broadcast instance, each non-faulty process p_i broadcasts a binary value and obtains (BV-delivers) a set of binary values, stored in a local read-only set variable denoted bin_values_i . This set, initialized to \emptyset , increases when new values are received. BV-broadcast is defined by the four following properties:

- BV-Obligation. If at least $(t + 1)$ non-faulty processes BV-broadcast the same value v , v is eventually added to the set bin_values_i of each non-faulty process p_i .
- BV-Justification. If p_i is non-faulty and $v \in bin_values_i$, v has been BV-broadcast by a non-faulty process.
- BV-Uniformity. If a value v is added to the set bin_values_i of a non-faulty process p_i , eventually $v \in bin_values_j$ at every non-faulty process p_j .
- BV-Termination. Eventually the set bin_values_i of each non-faulty process p_i is not empty.

The following property is an immediate consequence of the previous properties. Eventually the sets bin_values_i of the non-faulty processes p_i (i) become non-empty, (ii) become equal, (iii) contain all the values broadcast by non-faulty processes, and (iv) never contain a value broadcast only by Byzantine processes. However, no non-faulty process knows when (ii) and (iii) occur.

3.2 Local variables and message types

Each process p_i manages the following local variables.

- est_i : local current estimate of the decided value. It is initialized to the value proposed by p_i .
- r_i : local asynchronous round number, initialized to 0.
- $bin_values_i[1..]$: array of binary values; $bin_values_i[r]$ (initialized to \emptyset) stores the local output set filled by BV-broadcast associated with round r . (This unbounded array can be replaced by a single local variable bin_values_i , reset to \emptyset at the beginning of every round. We consider here an array to simplify the presentation.)

- b_i : auxiliary binary value.
- $values_i$: auxiliary set of values.

The algorithm uses two message types, denoted EST and AUX. Both are used in each round, hence they always appear with a round number.

- EST[r]() is used at round r by p_i to BV-broadcast its current decision estimate est_i .
- AUX[r]() is used by p_i to disseminate its current value of $bin_values_i[r]$ (with the help of the broadcast() macro-operation).

3.3 A safe asynchronous binary Byzantine consensus algorithm

For the sake of simplicity, we first introduce a new leaderless algorithm ensuring BBC-Validity and BBC-Agreement properties in the system model $\mathcal{BAMP}_{n,t}[t < n/3]$ but not BBC-termination. The algorithm is depicted in Figure 1 and provides the process p_i with the operation bin_propose(v_i) to propose its initial value v_i . Process p_i proceeds in asynchronous rounds and decides value v when invoking decide(v) at line 10.

```

operation bin_propose( $v_i$ ) is
(01)   $est_i \leftarrow v_i; r_i \leftarrow 0;$ 
(02)  while (true) do
(03)     $r_i \leftarrow r_i + 1;$ 
(04)    BV_broadcast EST[ $r_i$ ]( $est_i$ ); // add to bin_values[ $r_i$ ] upon BV_delivery
(05)    wait_until ( $bin\_values_i[r_i] \neq \emptyset$ );
(06)    broadcast AUX[ $r_i$ ]( $bin\_values_i[r_i]$ );
(07)    wait_until (messages AUX[ $r_i$ ]( $b\_val_{p(1)}$ ), ..., AUX[ $r_i$ ]( $b\_val_{p(n-t)}$ ) have been received
                    from  $(n-t)$  different processes  $p(x)$ ,  $1 \leq x \leq n-t$ , and their contents are
                    such that  $\exists$  a non-empty set  $values_i$  where (i)  $values_i = \cup_{1 \leq x \leq n-t} b\_val_{p(x)}$ 
                    and (ii)  $values_i \subseteq bin\_values_i[r_i]$ );
(08)     $b_i \leftarrow r_i \bmod 2;$ 
(09)    if ( $values_i = \{v\}$ ) //  $values_i$  is a singleton whose element is  $v$ 
(10)      then  $est_i \leftarrow v$ ; if ( $v = b_i$ ) then decide( $v$ ) if not yet done end if;
(11)      else  $est_i \leftarrow b_i$ 
(12)      end if;
(13)  end while.

(14)  when B-VAL[ $r$ ]( $v$ ) is BV-delivered by BV_broadcast[ $r$ ] do
         $bin\_values_i[r] \leftarrow bin\_values_i[r] \cup \{v\};$ 

```

Figure 1: A safe algorithm for the binary Byzantine consensus in $\mathcal{BAMP}_{n,t}[t < n/3]$

After it has deposited its binary proposal in est_i (line 01), each non-faulty process p_i enters a sequence of asynchronous rounds. During a round r , each non-faulty process p_i proceeds in three phases.

Phase 1: Binary value broadcast to filter out the values of Byzantine processes. Process p_i first progresses to the next round, and binary value broadcasts (BV-broadcast) its current estimate (line 04).

At each process p_i , within the $BV_broadcast()$ algorithm, after receiving the same value from $t + 1$ processes, process p_i then rebroadcasts this value. Each process p_i BV-delivers a value v by adding it to its bin_values_i set only if it receives v from $2t + 1$ distinct processes. Eventually the sets bin_values of all non-faulty processes become non-empty, equal, and contain exclusively all values broadcast by non-faulty processes [19]. When a value is BV-delivered it is then added to $bin_values_i[r]$ (line 14). Then p_i waits until its set $bin_values_i[r]$ is not empty (let us recall that, when $bin_values_i[r]$ becomes non-empty, it has not necessarily its final value).

Phase 2: Exchanging estimates to converge to an agreement. This second phase runs between line 06 and line 07). In this phase, p_i broadcasts normally a message $AUX[r]()$ whose content is $bin_values_i[r]$ (line 06). Then, p_i waits until it has received a set of values $values_i$ satisfying the two following properties.

- The values in $values_i$ come from the messages $AUX[r]()$ of at least $(n - t)$ different processes.
- $values_i \subseteq bin_values_i[r]$. Thanks to the BV-broadcast that filters out Byzantine value, even if Byzantine processes send fake messages $AUX[r]()$ containing values proposed only by Byzantine processes, $values_i$ will contain only values broadcast by non-faulty processes.

Hence, at any round r , after line 07, $values_i \subseteq \{0, 1\}$ and contains only values BV-broadcast at line 04 by non-faulty processes.

Phase 3: Deciding upon estimate convergence to round number modulo 2. The third phase runs between line 08 and line 12. This phase is a purely local computation phase, during which (if not yet done) p_i tries to decide the value $b = r \bmod 2$ (lines 08 and 10), depending on the content of $values_i$.

- If $values_i$ contains a single element v (line 09), then v becomes p_i 's new estimate. Moreover, v is a candidate for the consensus decision. To ensure

BBC-Agreement, v can be decided only if $v = b$. The decision is realized by the statement `decide(v)` (line 10).

- If $values_i = \{0, 1\}$, then p_i cannot decide. As both values have been proposed by non-faulty processes, to entail convergence to agreement, p_i selects one of them (b , which is the same at all non-faulty processes for the same round) as its new estimate (line 11).

Let us observe that the invocation of `decide(v)` by p_i does not terminate the participation of p_i in the algorithm, namely p_i continues looping forever. This is because a deciding process may need to help other processes converging to the decision in the two subsequent rounds. This algorithm can be modified to avoid this infinite loop, but to preserve the simplicity in the presentation, we postpone a deterministic terminating solution to Section 3.4. The proof of correctness of algorithm 1 is deferred to the appendix.

3.4 Psync: Safe and Live Consensus in $\mathcal{BAMP}_{n,t}[t < n/3, \diamond Synch]$

We now present *Psync*, an algorithm solving the binary Byzantine consensus problem in the $\mathcal{BAMP}_{n,t}[t < n/3, \diamond Synch]$ model. Similar to the safe algorithm (Section 3.3), *Psync* does not use signatures or randomization and has the following additional characteristics:

- *Psync* is time optimal [23] in that it terminates in $O(t)$ message delays.
- When all non-faulty processes propose the same value, *Psync* terminates in $O(1)$ message delays, even under asynchrony.
- *Psync* does not wait for a message from its coordinator and does not need recovery.

The *Psync* algorithm is presented in Figure 2 as an extension of the safe algorithm in Figure 1, with new and modified lines prefixed with “New” and “M-”, respectively. Lines prefixed by “Opt” are optional optimizations. In addition to the use of local timers, to eventually benefit from the $\diamond Synch$ assumption, the algorithm uses a *weak coordinator*: the weak coordinator of round r is the process p_i such that $i = ((r - 1) \bmod n) + 1$. Note that this new round coordinator is only used to help agreement by suggesting a value and thus differs from the classic coordinator [17, 22].

Additional local variables and message type. In addition to est_i , r_i , $bin_values_i[r]$, and $values_i$, each process p_i manages the following local variables.

```

operation bin_propose( $v_i$ ) is
(01)   $est_i \leftarrow v_i$ ;  $r_i \leftarrow 0$ ;
       $timeout_i \leftarrow 0$ ;
(02)  while (true) do
(03)     $r_i \leftarrow r_i + 1$ ;
(Opt1) if ( $est_i = -1$ ) then  $est_i \leftarrow 1$ ; // "fast-path" for round 1, only used in the reduction in Sect. 4
(04)    else BV_broadcast EST[ $r_i$ ]( $est_i$ );
      end if;
(New1) wait_until ( $bin\_values_i[r_i] \neq \emptyset$ );
       $timeout_i \leftarrow timeout_i + 1$ ; set timer $_i$  to  $timeout_i$ ;
(New2)  $coord_i \leftarrow ((r_i - 1) \bmod n) + 1$ ;
      if ( $i = coord_i$ ) then
         $\{w\} = bin\_values_i[r_i]$ ; //  $w$  is the first value to enter  $bin\_values_i[r_i]$ 
        broadcast COORD_VALUE[ $r_i$ ]( $w$ )
      end if;
(M-05) wait_until ( $(bin\_values_i[r_i] \neq \emptyset) \wedge (timer_i \text{ expired})$ );
(New3) if ( $(COORD\_VALUE[r_i](w) \text{ received from } p_{coord_i}) \wedge (w \in bin\_values_i[r_i])$ )
      then  $aux_i \leftarrow \{w\}$ 
      else  $aux_i \leftarrow bin\_values_i[r_i]$ 
      end if;
(M-06) broadcast AUX[ $r_i$ ]( $aux_i$ );
(New4) wait_until (a message AUX[ $r_i$ ]() has been received from  $(n - t)$  different processes);
      set timer $_i$  to  $timeout_i$ ;
(M-07) wait_until ( $(\text{messages } AUX[r_i](b\_val_{p(1)}), \dots, AUX[r_i](b\_val_{p(n-t)}) \text{ have been received}$ 
      from  $(n - t)$  different processes  $p(x)$ ,  $1 \leq x \leq n - t$ , and their contents are
      such that  $\exists$  a non-empty set  $values_i$  where (i)  $values_i = \cup_{1 \leq x \leq n-t} b\_val_{p(x)}$ 
      and (ii)  $values_i \subseteq bin\_values_i[r_i] \wedge (timer_i \text{ expired})$ );
(New5) if (when considering the whole set of the messages AUX[ $r_i$ ]() received, several sets
       $values_{1i}, values_{2i}, \dots$  satisfy the previous wait predicate)  $\wedge$  (one of them is  $aux_i$ )
      then  $values_i \leftarrow aux_i$  end if; //  $values_i$  is either defined here or at line M07
(08)   $b_i \leftarrow r_i \bmod 2$ ;
(09)  if ( $values_i = \{v\}$ ) //  $values_i$  is a singleton whose element is  $v$ 
(10)    then  $est_i \leftarrow v$ ; if ( $v = b_i$ ) then decide( $v$ ) if not yet done end if;
(11)    else  $est_i \leftarrow b_i$ 
(12)    end if;
(Opt2) if (decided in round  $r_i$ ) then // the following are termination conditions
      wait until ( $bin\_values_i[r_i] = \{0, 1\}$ ) // only go to the next round when necessary
      else if (decided in round  $r_i - 2$ ) then halt end if; // everyone has decided by now
      end if;
(13) end while.

```

Figure 2: A safe and live algorithm for the binary Byzantine consensus in $BAMP_{n,t}[t < n/3, \diamond Synch]$; line (Opt1) is an optimization only applied in the multivalued reduction presented in Section 4; line (Opt2) is a mechanism to prevent unnecessary rounds from being executed

- $timer_i$ is a local timer, and $timeout_i$ a timeout value, both used to exploit the assumption $\diamond Synch$.
- $coord_i$ is the index of the current weak round coordinator.
- aux_i is an auxiliary set of values, used to store the value (if any) that the current weak coordinator strives to impose as decision value.

The weak coordinator of round r , uses the message type $COORD_VALUE[r]()$ to broadcast the value it suggests for decision.

Description of the extended algorithm. We now list the new and modified lines that were added in Figure 2.

- At line New1, p_i waits until a value enters bin_values , then sets its local timer, whose expiry is used in the predicate of line M-05. The timeout value is initialized before entering the loop, and then increased at every round.
- Line Opt1 is an optimization only used along with the reduction to multivalued consensus presented in Section 4.
- Line New4 waits until $(n-t)$ $AUX[r]()$ messages are received from different processes before resetting the timer, whose expiry is used in the predicate of the modified line M-07.
- Lines New2, New3, M-06, and New5 realize a mechanism that allows the current weak coordinator (whose value is computed on line New2) to try to impose the first value that enters into its bin_values set as the decided value. Combined with the fact that there is a time after which the messages exchanged by the non-faulty processes are timely, this ensures that there will be a round during which the non-faulty processes will have a single value in their sets $values_i$, which entails their decision.
- Modified lines M-05 and M-07: addition of the timer expiration in the predicate considered at the corresponding line.
- Line Opt2 is an optional optimization to minimize the amount of extra rounds processes need to execute after deciding. The first condition (**wait until** ($bin_values_i[r_i] = \{0, 1\}$)) ensures that, after decision, a process only continues to the next round if some other non-faulty process did not decide in the current round. As this can only happen if both 0 and 1 enter bin_values , the process will not move on to the next round until this is true. The second condition, (**if** (decided in round $r_i - 2$)), halts the process 2 rounds after it has decided, as all non-faulty processes are guaranteed to have decided by this round.

The aforementioned modifications exploit the weak coordinator that only helps resolving disagreement by broadcasting a value that all non-faulty

adopt, as opposed to leaders or classic (strong) coordinators [17,22]. To this end:

- The weak coordinator p_k broadcasts the message $\text{COORD_VALUE}[r_i](w)$, where w is the first value that enters its bin_values set (line New2). If p_k is non-faulty, the timeout values of the non-faulty processes are big enough, and there is a bound on message transfer delays, so that all non-faulty processes will receive it before their timer expiration at line M-05.
- Then, assuming the previous item, all non-faulty processes set aux_i to $\{w\}$ (line New3), and broadcast it (line M-06). The predicate $w \in \text{bin_values}_i[r_i]$ is used to prevent a Byzantine coordinator to send fake values that would foil non-faulty processes.
- Finally, all the non-faulty processes will receive the message $\text{AUX}[r_i](\{w\})$ from $(n-t)$ different processes, and, by line New5, will set $\text{values}_i = \{w\}$. This entails their decision during the round $(r+1)$ or $(r+2)$.

To ensure that slow processes catch up to faster processes that have reached later rounds, once a process has received at least $t+1$ messages belonging to a round r , the process does wait for timeouts in rounds less than r . In the presence of $\diamond\text{Synch}$, this ensures that all non-faulty processes eventually execute synchronous rounds. The proof of liveness of algorithm 2 is deferred to the appendix.

4 DBFT: From Binary Byzantine Consensus to Blockchain Consensus

This section presents a *Democratic Binary Fault Tolerant* algorithm, called DBFT. It relies on a reduction from the binary Byzantine consensus Psync to the multivalued consensus and is also time optimal, resilience optimal and does not use classic (strong) coordinator, which means that it does not wait for a particular message. In addition, it finishes in only 4 message delays in the good case, when all non-faulty processes propose the same value.

We consider a variant of the classical Byzantine consensus problem, called the *Validity Predicate-based Byzantine Consensus* (denoted VPBC). Its validity requirement relies on an application-specific $\text{valid}()$ predicate that is used by blockchains to indicate whether a value is *valid*. Assuming that each non-faulty process proposes a valid value, each of them has to decide on a value in such a way that the following properties are satisfied.

- VPBC-Termination. Every non-faulty process eventually decides on a value.

- VPBC-Agreement. No two non-faulty processes decide on different values.
- VPBC-Validity. A decided value is valid, i.e., it satisfies the predefined predicate denoted $\text{valid}()$, and if all non-faulty processes propose the same value v then they decide v .

This definition generalizes the classical definition of Byzantine consensus, which does not include the predicate $\text{valid}()$. This predicate is introduced to take into account the distinctive characteristics of consortium blockchains, and possibly other specific Byzantine consensus problems. In the context of consortium blockchains, a proposal is not valid if it does not contain an appropriate hash of the last block added to the Blockchain or contains invalid transactions. There exist similar problem definitions whose validity also relies on the notion of a predicate. The validated Byzantine consensus [13] differs in that the same valid value proposed by non-faulty processes has to be decided if all processes are non-faulty. The asynchronous Byzantine agreement [29] defines a legal value similar to our valid value, however, its validity does not require a legal value to be decided if multiple ones exist, while we require that any decided value must be valid. A probabilistic variant [14] required that the decided value be one of the proposed values, something we do not require.

```

operation mv_propose( $v_i$ ) is
(01) RB_broadcast VAL( $v_i$ );
(02) repeat if ( $\exists k : (\text{proposals}_i[k] \neq \perp) \wedge$ 
      ( $\text{BIN\_CONS}[k].\text{bin\_propose}()$  not invoked))
(03)       then invoke  $\text{BIN\_CONS}[k].\text{bin\_propose}(-1)$  end if;
(04) until ( $\exists \ell : \text{bin\_decisions}_i[\ell] = 1$ ) end repeat;
(05) for each  $k$  s.t.  $\text{BIN\_CONS}[k].\text{bin\_propose}()$  not yet invoked
(06)   do invoke  $\text{BIN\_CONS}[k].\text{bin\_propose}(0)$  end for;
(07) wait_until ( $\bigwedge_{1 \leq x \leq n} \text{bin\_decisions}_i[x] \neq \perp$ );
(08)  $j \leftarrow \min\{x \text{ such that } \text{bin\_decisions}_i[x] = 1\}$ ;
(09) wait_until ( $\text{proposals}_i[j] \neq \perp$ );
(10) decide( $\text{proposals}_i[j]$ ).

(11) when VAL( $v$ ) is RB-delivered from  $p_j$  do
      if valid( $v$ ) then
         $\text{proposals}_i[j] \leftarrow v$ ;
        BV-deliver B-VAL[1](1) to  $\text{BIN\_CONS}[j]$  end if.

(12) when  $\text{BIN\_CONS}[k].\text{bin\_propose}()$  decides a value  $b$ 
      do  $\text{bin\_decisions}_i[k] \leftarrow b$ .

```

Figure 3: From multivalued to binary Byzantine consensus in $\mathcal{BAMP}_{n,t}[t < n/3, \text{BBC}]$

Binary consensus objects. The processes cooperate with an array of binary Byzantine consensus objects denoted $BIN_CONS[1..n]$. The instance $BIN_CONS[k]$ allows the non-faulty processes to find an agreement on the value proposed by p_k . This object is implemented with the binary Byzantine consensus algorithm presented in Section 3.4. To simplify the presentation, we consider that a process p_i launches its participation in $BIN_CONS[k]$ by invoking $BIN_CONS[k].bin_propose(v)$, where $v \in \{0, 1\}$. Then, it executes the corresponding code in a specific thread, which eventually returns the value decided by $BIN_CONS[k]$.

Local variables. Each process p_i manages the following local variables; \perp denotes a default value that cannot be proposed by a (faulty or non-faulty) process.

- An array $proposals_i[1..n]$ initialized to $[\perp, \dots, \perp]$. The aim of $proposals_i[j]$ is to contain the value proposed by p_j .
- An array $bin_decisions_i[1..n]$ initialized to $[\perp, \dots, \perp]$. The aim of $bin_decisions_i[k]$ is to contain the value (0 or 1) decided by the binary consensus object $BIN_CONS[k]$.

The algorithm. The algorithm reducing from the binary Byzantine consensus to multivalued Byzantine consensus is described in Figure 3 and is similar to an existing reduction [7], except that it combines the reliable broadcast, RB-broadcast [10], restated in the appendix, with our binary consensus messages to finish in 4 message delays in the good case. Initially, a process invokes the operation $mv_propose(v)$, where v is the value it proposes to the multivalued consensus. Process p_i executes four phases.

Phase 1: p_i disseminates its value (lines 01 and 11). Process p_i first sends its value to all the processes by invoking the RB-broadcast operation (line 01). If a process RB-delivers a valid value v RB-broadcast by a process p_j , then the process stores it in $proposals_i[j]$ and BV-delivers 1 directly to round one of instance $BIN_CONS[j]$ (line 11), placing 1 in its bin_values_i for that instance.

Phase 2: Process p_i starts participating in a first set of binary consensus instances (lines 02-04). It enters a loop in which it starts participating in the binary consensus instances. Process p_i invokes a binary consensus instance k with value -1 for each value RB-broadcast by process p_k that p_i RB-delivered. -1 is a special value that allows the binary consensus to skip the BV_broadcast step (line (Opt1)) and immediately send an AUX message with value 1, allowing the binary consensus to terminate with value 1 in a single

message delay. (Note that the timeout of the first round is set to 0 so the binary consensus proceeds as fast as possible.) The direct delivery of 1 into *bin_values* is possible due to an overlap in the properties of BV_broadcast and RB-broadcast, allowing us to skip a message step of our binary consensus algorithm. In other words, all non-faulty processes will RB-deliver the proposed value, and as a result will also BV-deliver 1. This loop stops as soon as p_i discovers a binary consensus instance $BIN_CONS[\ell]$ in which 1 was decided (line 04). (As all non-faulty processes will only have 1 in their *bin_values* until an instance terminates, the first instance to decide 1 will terminate in one message delay following the RB-delivery.)

Phase 3: p_i starts participating in all other binary consensus instances (lines 05-06). After it knows a binary consensus instance decided 1, p_i invokes with `bin_propose(0)` all the binary consensus instances $BIN_CONS[k]$ in which it has not yet participated. Let us notice that it is possible that, for some of these instances $BIN_CONS[k]$, no process has RB-delivered a value from the associated process p_k . The aim of these consensus participation is to ensure that all binary consensus instances eventually terminate.

Phase 4: p_i decides a value (lines 07-10 and 12). Process p_i considers the first (according to the process index order) among the successful binary consensus objects, i.e., the ones that returned 1 (line 08). Let $BIN_CONS[j]$ be this binary consensus object. As the associated decided value is 1, at least one non-faulty process proposed 1, which means that it RB-delivered a value from the process p_j (lines 02-03). Observe that this value is eventually RB-delivered by every non-faulty process. Consequently, p_i decides it (lines 09-10). Notice that as soon as the binary consensus instance with the smallest process index terminates with 1, the reduction can return as soon as the associated value is RB-delivered. This is due to the observation that the values associated with the larger indices will not be used.

Complexity. This eager termination allows the consensus algorithm to terminate in 4 message delays in the good scenario, i.e., 3 message delays to execute the reliable broadcast and 1 to complete the binary consensus by skipping the BV_broadcast step. In this case the reliable broadcast and binary consensus each have $O(n^2)$ message complexity for a total of $O(n^3)$ including all n instances. In the case of faulty processes or asynchrony the algorithm will need at least 3 additional message delays for binary consensus instances to terminate with 0.

Theorem 1. *The algorithm described in Figure 3 implements the multi-valued Byzantine consensus (VPBC) in the system model $\mathcal{BAMP}_{n,t}[t <$*

$n/3$, BBC].

The proof of correctness of DBFT is deferred to the appendix.

5 Conclusion

To conclude, our weak coordinator based Byzantine consensus is time optimal, resilience optimal, does not rely on randomization or signatures and improves over the randomized Byzantine consensus algorithms [37, 38] by terminating faster in various geo-distributed experiments. We presented how it can be used for consortium blockchains by generalizing the Byzantine consensus problem and presenting a solution that combines an existing reduction with our binary Byzantine consensus algorithm.

DBFT is now at the heart of the Red Belly Blockchain, a fast permissioned blockchain. Future work involves extending this permissioned blockchain into a public blockchain using DBFT for reconfiguration to periodically change at runtime the subset of machines running the consensus, similar to Solida [1] but without proof-of-work.

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A Experiments on 100 VMs on Distinct Continents

In this section, we evaluate the performance of our consensus algorithm against a randomized consensus applied to blockchains on 100 Amazon machines located in 5 distinct data centers across different continents.

A.1 Experimental setup

To measure the performance of our consensus algorithm in a real network setting, we deployed our binary consensus algorithm called “Psync” on 100 machines distributed across different continents.

To implement point-to-point reliable channels over the Internet, we implemented secure channels using TLS on top of TCP/IP. Note that TLS uses a public key cryptosystem (and signatures) only to exchange secret keys, but no signatures are used by our consensus algorithm. Note that the Red Belly Blockchain builds upon the same combination of DBFT and TLS by storing the necessary certificates in its blocks [25].

For the sake of comparison, we also implemented the randomized binary Byzantine consensus algorithm from Mostéfaoui et al. [37], called “Coin”, as a baseline. Coin terminates in $O(1)$ rounds in expectation and is at the heart of the HoneyBadger permissioned blockchain [36] but requires a fair scheduler [38]. Our implementation reuses the common coin implementation of HoneyBadger [36] that consists of a one step message exchange and threshold signatures. All 100 machines are c4.xlarge of Amazon EC2 equipped with an Intel Xeon E5-2666 v3 with 4 vCPUs, 7.5 GiB RAM, and “moderate” network performance.

We set the timeouts of Psync to be null in the first t rounds before incrementing exponentially. We implemented reliability using sequence numbers and negative acknowledgments at the application level. All consensus decisions are stored to disk in an append only log. Results are taken as the average of 100 instances of consensus.

A.2 Geo-distributed experiments between 5 datacenters

Figure 4 compares the average latency and number of rounds needed to terminate Psync and Coin in 5 Amazon datacenters, 3 in the US (Oregon, Northern California, and Ohio) and 2 in Europe (Ireland and Frankfurt). Our ping latency across continents is between 91 ms and 164 ms and within one continent between 22 ms and 71 ms. In Figure 4(left) the x-axis denotes

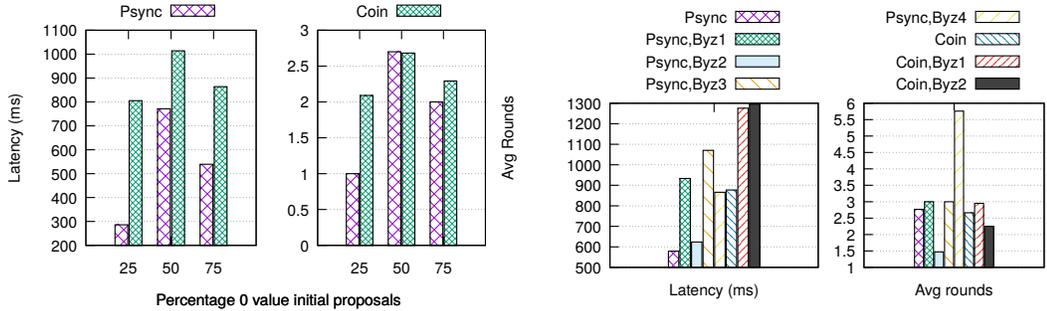


Figure 4: Geo-distributed experiments of latency and average number of rounds for our deterministic binary Byzantine consensus “Psync” and the randomized binary Byzantine consensus “Coin” with 4 different Byzantine attacks (Byz1, ..., Byz4): **(left)** with varying levels of initial disagreement, **(right)** with random initial proposals.

the approximate percentage of processes that have an initial proposal of 0 (others proposing 1). Psync terminates in at most three rounds on average.

Given that Psync is designed to terminate with 1 in the first round and 0 in the second round, the best performance is reached when the majority of proposals are 1. In all cases the latency of Psync is lower than Coin due to the coin needing an extra message step, additional computation complexity, and randomness.

A.3 Tolerance to various Byzantine attacks

Figure 4(right) compares the algorithms with the following Byzantine behaviors: (Byz1) Byzantine processes flip the binary values of their messages; (Byz2) Byzantine processes are mute; (Byz3) extends Byz1 with Byzantine coordinators that send random binary values in their `COORD_VALUE` messages; (Byz4) Byzantine processes form a coalition to limit the progress within rounds by sending their own messages without waiting so they can be processed before others. Both Byz3 and Byz4 are specific to Psync.

More precisely, Byz4 mimics a behavior where the coordinator is faulty to limit progress during rounds by trying to have *(i)* no non-faulty processes to decide in round r and *(ii)* have two non-faulty processes starting round $r + 1$ with distinct estimates. To this end, the faulty nodes start the round by broadcasting both 1 and 0 in their BV-broadcast. Then, the Byzantine coordinator sends a message `coord_value` with $\neg(r \bmod 2)$ to all non-faulty nodes. Finally, Byzantine nodes instantly send `AUX` message with value

$\neg(r \bmod 2)$ to a single node and send AUX message with value $(r \bmod 2)$ to the remaining nodes. Faulty nodes in Byz4 have the power to send their messages instantly and to observe the messages received at non-faulty nodes, giving them more power to delay termination. They do not control the speed or order of messages from non-faulty nodes.

In Psync, the Byzantine processes are chosen as the first t coordinators. Coin has the highest latency with the Byzantine behaviors, but its number of rounds is least affected. Byzantine behavior Byz3 is the slowest to terminate for Psync because it allows Byzantine processes to force the most disagreement. While theoretically Byz4 could always prevent termination in the first t rounds, the average number of rounds is only increased to 6 (but has a maximum of 35). This is due to the fact that they do not control the speed of messages of non-faulty processes in the network preventing the non-terminating case. Furthermore, given that the Byzantine processes have to act fast to ensure their messages are processed first, the average latency is lower than Byz1 and Byz2.

A.4 Detailed description of Byzantine behavior Byz4

In the presence of a faulty coordinator it is possible to execute repeated rounds in which there is no termination, behavior Byz4 tries to capture this behavior. Note that we allow Byzantine messages to be delivered instantly by computing them directly at the non-faulty nodes when needed. We will now describe the Byz4 behavior. Assume we are in a round r . There are two main things we need to ensure: (i) no non-faulty process decides in round r (ii) at least one non-faulty node must start round $r + 1$ with an estimate of 0 and another start with the estimate of 1.

To ensure (i) we need (a) $\neg(r \bmod 2)$ to enter *bin_values* of non-faulty nodes and (b) no node must receive $n-t$ AUX messages with value $(r \bmod 2)$. Then to ensure (ii) we need (c) both 0 and 1 to enter *bin_values* of non-faulty nodes, (d) at least one node must receive receive $n - t$ AUX messages with value $\neg(r \bmod 2)$, and (e) at least one node must receive receive an AUX messages with value $(r \bmod 2)$.

Thus, Byzantine nodes start the round by broadcasting both 1 and 0 in their BV-broadcast to ensure (a) and (c). To try to ensure (b), the Byzantine coordinator sends a message *coord_value* with $\neg(r \bmod 2)$ to all non-faulty nodes, this message is delivered instantly, as a result all non-faulty processes broadcast an AUX message with value $\neg(r \bmod 2)$. Then to ensure (d), Byzantine nodes instantly send AUX message with value $\neg(r \bmod 2)$ to a single node. Furthermore, to ensure (e), Byzantine nodes instantly send AUX

message with value $(r \bmod 2)$ to the remaining nodes. Assuming both 0 and 1 entered bin_values at appropriate times at non-faulty nodes, termination will be prevented for this round.

The difficulty in ensuring this non-termination scenario is that the Byzantine nodes do not control the time that both 1 and 0 enter bin_values of non-faulty nodes. If $\neg(r \bmod 2)$ enters too late, a process may broadcast $(r \bmod 2)$ as its AUX message, and as a result we may fail with (d). Otherwise if $(r \bmod 2)$ enters bin_values too late, all non-faulty processes may terminate with $n - t$ AUX messages with value $\neg(r \bmod 2)$. Similar timing arguments can be made for other non-terminating scenarios that use different message patterns.

A.5 Different experiment configurations

Figure 5 uses the same experimental settings as Figure 4, except is run with 100 nodes within a single datacenter. Here we see a much larger gap in latency between Psync and Coin as the computation of the cryptographic operations of the random coin is much larger than the network latency. Note that the latency of both algorithms could be decreased through the use of message authentication codes (MACs) with datagram broadcasts, but we expect the latency to still be dominated by the cryptotgraphic operations of the coin.

Figure 5 uses the same experimental settings as Figure 4, except is run with 1 node in each of Amazon’s 14 EC2 data centers. The results are similar to the 5 datacenter case of Figure 4, but with higher latency in most cases due to the increased geo-distribution.

B Proofs of safety and liveness of the algorithms

B.1 Safety proof of the binary Byzantine consensus (Figure 1)

The proof is described from a point of view of a non-faulty process p_i . Let $values_i^r$ denote the value of the set $values_i$ which satisfies the predicate of line 07 during a round r . Moreover, let us recall that, given a run, C denotes the set of non-faulty processes in this run.

Lemma 1. *Let $t < n/3$. If at the beginning of a round r , all non-faulty processes have the same estimate v , they never change their estimate value thereafter.*

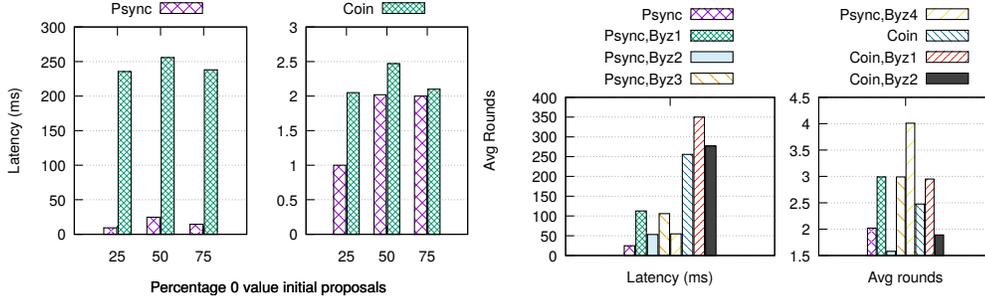


Figure 5: Single datacenter comparison of latency and average number of rounds to terminate of our deterministic binary Byzantine consensus against randomized binary Byzantine consensus: **(Left)**

Proof Let us assume that all non-faulty processes (which are at least $n-t > t+1$) have the same estimate v when they start round r . Hence, they all BV-broadcast the same message $\text{EST}[r](v)$ at line 04. It follows from the BV-Justification and BV-Obligation properties that each non-faulty process p_i is such that $\text{bin_values}_i[r] = \{v\}$ at line 05, and consequently can broadcast only $\text{AUX}[r](\{v\})$ at line 06. Considering any non-faulty process p_i , it then follows from the predicate of line 07 (values_i contains only v), the predicate of line 09 (values_i is a singleton), and the assignment of line 10, that est_i keeps the value v . $\square_{\text{Lemma 1}}$

Lemma 2. Let $t < n/3$. $((p_i, p_j \in C) \wedge (\text{values}_i^r = \{v\}) \wedge (\text{values}_j^r = \{w\})) \Rightarrow (v = w)$.

Proof Let p_i be a non-faulty process such that $\text{values}_i^r = \{v\}$. It follows from line 07 that p_i received the same message $\text{AUX}[r](\{v\})$ from $(n-t)$ different processes, i.e., from at least $(n-2t)$ different non-faulty processes. As $n-2t \geq t+1$, this means that p_i received the message $\text{AUX}[r](\{v\})$ from a set Q_i including at least $(t+1)$ different non-faulty processes.

Let p_j be a non-faulty process such that $\text{values}_j^r = \{w\}$. Hence, p_j received $\text{AUX}[r](\{w\})$ from a set Q_j of at least $(n-t)$ different processes. As $(n-t)+(t+1) > n$, it follows that $Q_i \cap Q_j \neq \emptyset$. Let $p_k \in Q_i \cap Q_j$. As $p_k \in Q_i$, it is a non-faulty process. Hence, at line 06, p_k sent the same message $\text{AUX}[r](\{v\})$ to p_i and p_j , and we consequently have $v = w$. $\square_{\text{Lemma 2}}$

Lemma 3. *Let $t < n/3$. The value decided by a non-faulty process was proposed by a non-faulty process.*

Proof Let us consider the round $r = 1$. Due to the BV-Justification property of the BV-broadcast of line 04, it follows that the sets $bin_values_i[1]$ contains only values proposed by non-faulty processes. Consequently, the non-faulty processes broadcast at line 06 messages $AUX[1]()$ containing sets with values proposed only by non-faulty processes. It then follows from the predicate (i) of line 07 ($values_i^1 \subseteq bin_values_i[1]$), and the BV-Justification property of the BV-broadcast abstraction, that the set $values_i^1$ of each non-faulty process contains only values proposed by non-faulty processes. Hence, the assignment of est_i (be it at line 10 or 11) provides it with a value proposed by a non-faulty process. The same reasoning applies to rounds $r = 2, r = 3$, etc., which concludes the proof of the lemma. $\square_{Lemma\ 3}$

Lemma 4. *Let $t < n/3$. No two non-faulty processes decide different values.*

Proof Let r be the first round during which a non-faulty process decides, let p_i be a non-faulty process that decides in round r (line 10), and let v be the value it decides. Hence, we have $values_i^r = \{v\}$ where $v = (r \bmod 2)$.

If another non-faulty process p_j decides during round r , we have $values_j^r = \{w\}$, and, due to Lemma 2, we have $w = v$. Hence, all non-faulty processes that decide in round r , decide v . Moreover, each non-faulty process that decides in round r has previously assigned $v = (r \bmod 2)$ to its local estimate est_i .

Let p_j be a non-faulty that does not decide in round r . As $values_i^r = \{v\}$, and p_j does not decide in round r , it follows from Lemma 2 that we cannot have $values_j^r = \{1 - v\}$, and consequently $values_j^r = \{0, 1\}$. Hence, in round r , p_j executes line 11, where it assigns the value $(r \bmod 2) = v$ to its local estimate est_j .

It follows that all non-faulty processes start round $(r + 1)$ with the same local estimate $v = r \bmod 2$. Due to Lemma 1, they keep this estimate value forever. Hence, no different value can be decided in a future round by a non-faulty process that has not decided during round r , which concludes the proof of the lemma. $\square_{Lemma\ 4}$

Lemma 5. *Let the system model be $\mathcal{BAMP}_{n,t}[t < n/3]$. No non-faulty process remains blocked forever in a round.*

Proof Let us assume by contradiction that there is a first round in which some non-faulty process p_i remains blocked forever. As all non-faulty processes terminate round $(r - 1)$, they all start round r and all invoke the

round r instance of BV-broadcast. Due to the BV-Termination property, the `wait_until()` statement of line 05 terminates at each non-faulty process. Then, as all non-faulty processes broadcast a message `AUX[r]()` (line 06), it follows that the `wait_until()` statement of line 07 terminates at each non-faulty process. It follows that there is no first round at which a non-faulty process remains blocked forever during round r . $\square_{\text{Lemma 5}}$

Lemma 6. *Let the system model be $\mathcal{BAMP}_{n,t}[t < n/3]$. If all non-faulty processes p_i terminate a round r with values $s_i^r = \{v\}$, they all decide by round $(r + 1)$.*

Proof If all non-faulty processes are such that $values_i^r = \{v\}$, and the round r is such that $v = (r \bmod 2)$, it follows from lines 08-10 that (if not yet done) each non-faulty process decides during round r .

If r is such that $v \neq (r \bmod 2)$, each non-faulty process sets its current estimate to v (line 10). As during the next round we have $v = ((r + 1) \bmod 2)$, and $values_i^{r+1} = bin_values_i[r + 1] = \{v\}$ at each non-faulty process p_i , each non-faulty process decides during round $(r + 1)$. $\square_{\text{Lemma 6}}$

Lemma 7. *Let the system model be $\mathcal{BAMP}_{n,t}[t < n/3]$. If every non-faulty process p_i terminates a round r with values $s_i^r = \{0, 1\}$, then it decides by round $(r + 2)$.*

Proof If every non-faulty process p_i is such that $values_i^r = \{0, 1\}$, it executes line 11 during round r , and we have $est_i = (r \bmod 2) = v$ when it starts round $(r + 1)$. Due to Lemma 1, it keeps this estimate forever. As all non-faulty processes execute rounds $(r + 1)$ and $(r + 2)$ (Lemma 5) and $v = ((r + 2) \bmod 2)$, we have $values_i^{r+2} = \{v\}$, at each non-faulty process p_i . It follows that each non-faulty process decides at line 10. $\square_{\text{Lemma 7}}$

Theorem 2. *The algorithm described in Figure 1 satisfies the safety consensus properties.*

Proof The proof follows from Lemma 3 (BBC-Validity) and Lemma 4 (BBC-Agreement). $\square_{\text{Theorem 2}}$

Decision The algorithm described in Figure 1 does not guarantee decision. This may occur for example when some non-faulty processes propose 0, the other non-faulty processes propose 1, and the Byzantine processes play

double game, each proposing 0 or 1 to each non-faulty process, so that it never happens that at the end of a round all non-faulty processes have either $values_i = \{0, 1\}$, or they all have $values_i = \{v\}$ with v either 0 or 1. In other words, if not all non-faulty processes propose the same initial value, Byzantine processes can make, round after round, some non-faulty processes have $values_i = \{0, 1\}$, while the rest of non-faulty processes have $values_i = \{v\}$, with $v \neq (r \bmod 2)$, avoiding them to decide.¹

B.2 Why the safe algorithm does not terminate with $\diamond Synch$

To circumvent the consensus impossibility [24] and find a terminating solution, one could be tempted to consider the $\mathcal{BAMP}_{n,t}[t < n/3, \diamond Synch]$ model and setting a timer, that increases in each round, by replacing line 05 in Figure 1 with a new line called “New1” and a modified line 05 called “M-05”:

```

...
(New1)  $timeout_i \leftarrow timeout_i + 1$ ; set  $timer_i$  to  $timeout_i$ ;
(M-05) wait_until  $((bin\_values_i[r_i] \neq \emptyset) \wedge (timer_i \text{ expired}))$ ;
...

```

In fact, this could seem sufficient to eventually give enough time for messages to be delivered. As we explain below, it would still be possible for a Byzantine process to wait depending on the timer of the current round to send a message $BVAL(v)$ to a non-faulty process early enough so that this non-faulty process receives the message before its local timer expires but too late for this non-faulty process to rebroadcast it and for other non-faulty processes to deliver it before their timers expire.

As an example, consider a counter-example of $n = 4$ processes among which $t = 1$ process is Byzantine that starts from a round r such that $r \bmod 2 = 1$ with non-faulty processes with estimates 0, 0 and 1. There is an execution leading to a round $r + 1$ where $(r + 1) \bmod 2 = 0$ and non-faulty processes have estimates 0, 1 and 1. The symmetric of this counter-example can then be used from round $r + 1$ where non-faulty processes have estimates 0, 1 and 1 to round $r + 2$ where non-faulty processes have estimates 0, 0 and 1. An infinite sequence alternating this counter-example and its symmetric example illustrates an infinite execution where no non-faulty process decides.

The counter-example is represented as a distributed execution in Figure 6, where p_1 , p_2 and p_3 are non-faulty processes while p_4 is a Byzantine process, as a distributed execution where time increases from left to right,

¹In the case of the randomized binary consensus algorithm of [38], the common coin guarantees termination with probability 1, because eventually the singleton value in $values_i$ will match the coin.

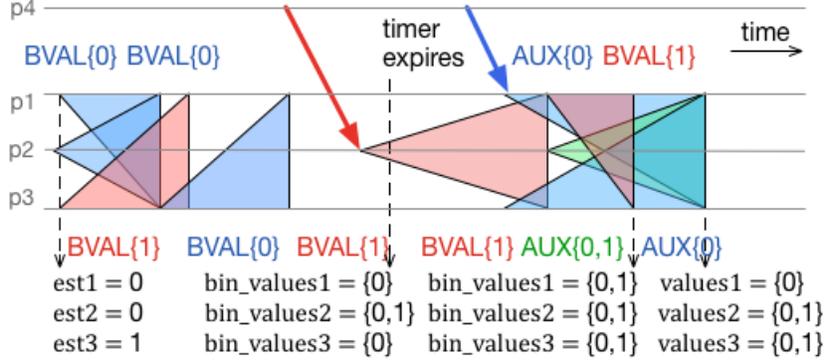


Figure 6: An execution of $n = 4$ processes exchanging broadcast messages (represented by triangles between $n - t = 3$ non-faulty processes) in round r ($r \bmod 2 = 1$) where $est_1 = est_2 = 0$ and $est_3 = 1$ leading to a round $r + 1$ where $est_1 = 0$ and $est_2 = est_3 = 1$

where arrows represent messages sent by the Byzantine process p_4 and triangles represent the broadcast messages among non-faulty processes: the left angle of each triangle indicates the source of the broadcast while the right edge indicates the processes where messages are delivered. (The receipt of messages by the Byzantine process p_4 are omitted for the sake of clarity in the presentation.)

The first four triangles represent the BV-broadcast (Figure 7) where p_1 and p_2 broadcasts $BVAL(0)$ while p_3 broadcasts $BVAL(1)$ according to their initial estimates. Once p_3 delivers $BVAL(0)$ from $t + 1 = 2$ non-faulty processes, it broadcasts the value 0 that it never broadcast before as specified in the code of Figure 7.

During BV-broadcast, all non-faulty processes receive from $2t + 1 = 3$ non-faulty processes. Now consider that the Byzantine process p_4 sends $BVAL(1)$ to p_2 which makes p_2 rebroadcast it as part of the BV-broadcast because it has now received message $BVAL(1)$ from $2 = t + 1$ distinct processes, namely p_3 and p_4 . We can thus obtain that $bin_values_1 = \{0\}$, $bin_values_2 = \{0, 1\}$ and $bin_values_3 = \{0\}$ at the time non-faulty processes broadcast their AUX messages. By sending $AUX(\{0\})$ to p_1 , the Byzantine process p_4 allows p_1 to choose $values_1 = \{0\}$ that has received $AUX(\{0\})$ from $n - t = 3$ distinct processes (p_1, p_3 and p_4). while the others have to choose $values_2 = values_3 = \{0, 1\}$ as they receive $AUX(\{0\})$, $AUX(\{0, 1\})$, $AUX(\{0\})$ from p_1, p_2 and p_3 , respectively. As $b = r \bmod 2 = 1$, it results

from line 11 that p_1 , p_2 and p_3 have estimates 0, 1, 1, respectively when starting the round $r + 1$.

Applying the symmetric example would lead to round $r + 2$ with the same estimates 0, 0, 1 as in round r , indicating the existence of an infinite execution.

B.3 Proof of Safety and Liveness of the \diamond_{Synch} -based Binary Byzantine Consensus (Figure 2)

The proof consists of two parts: (i) show that the added statements preserve the consensus safety properties proved for the algorithm of Figure 1, and (ii) show that all non-faulty processes eventually decide.

Lemma 8. *The algorithm described in Figure 2 satisfies the BBC-Validity and BBC-Agreement properties.*

Proof The proof consists in showing that the Lemmas 1, 2, 3 and 4 remain correct when considering the algorithm of Figure 2. Basically, these proofs remain correct because, as the new and modified statements do not assign values to the sets $bin_values_i[r]$ at the non-faulty processes, and no property of bin_values_i is related to a timing assumption, the set $bin_values_i[r]$ of a non-faulty process p_i can never contain values proposed by Byzantine processes only. It follows from this observation that the local variables est_i and $values_i$ of any non-faulty process p_i (defined or updated at lines M-07, New5, 10, or 11) can contain only values from non-faulty processes. More specifically we have the following.

- Lemma 1. Let r be the considered round, and v be the current estimate of the non-faulty processes. We then have $bin_values_i[r] = \{v\}$ at line M-05 of every non-faulty process p_i .
 - If the weak round coordinator p_k is non-faulty, we have at every non-faulty process $aux_i = bin_values_i[r] = \{v\}$. It then follows that $values_i^r = \{v\}$ and the lemma remains true due to lines 09 and 10.
 - If the weak round coordinator p_k is Byzantine and sends possibly different values to the non-faulty processes, let us consider a non-faulty process that receives the message $COORD_VALUE[r](\{1 - v\})$. As $(1 - v) \notin bin_values_i[r]$, at line New3, p_i executes the “else” part where it sets aux_i to $\{v\}$ (the only value in $bin_values_i[r]$), and the lemma follows.

- Lemma 2. As it does not depend on the timers, and is related only to the fact that each of the sets $values_i^r$ and $values_j^r$ of two non-faulty processes are singletons, the proof remains valid.
- Lemma 3. The proof follows from the fact that the sets bin_values_i of any non-faulty process can contain only values proposed by non-faulty processes.
- Lemma 4. As it relies only on the set $values_i^r$ of each non-faulty process p_i , this proof remains correct.

□_{Lemma 8}

Lemma 9. *The algorithm described in Figure 2 ensures that every non-faulty process decides.*

Proof Let us first observe that, as timers always expire, the “wait” statements (modified lines M-05 and M-07) always terminate, and consequently Lemma 5 remains true. The reader can also check that the proof of Lemma 6 remains valid.

It remains to show that there is eventually a round r at the end of which all non-faulty processes p_i have the same value w in their set variables ($values_i^r = \{w\}$) (from which decision follows due to Lemma 6) The proof shows that, due to (a) the eventual synchrony assumption, (b) the weak round coordinator mechanism, and (c) the messages $COORD_VALUE[]()$ sent by the weak round coordinators, there is a round r such that $values_i^r = \{w\}$ at each non-faulty process.

Let us consider a time τ from which (due to Lemma 11) the system behaves synchronously (the timeout values of all non-faulty processes are such that all the messages exchanged by the non-faulty processes arrive timely). Let r be the smallest round number coordinated by a non-faulty process p_k after τ . At line New2 of round r , p_k broadcasts $COORD_VALUE[r](w)$, being w the first value that enters its set $bin_values_k[r]$. The message $COORD_VALUE[r](w)$ is received timely by all non-faulty processes, that set aux_i to $\{w\}$ in line New3. Consequently, in line M-06 all non-faulty processes broadcast $AUX[r](\{w\})$, and receive in line M-07 $(n - t)$ $AUX[r](\{w\})$ messages from different processes, setting in line New5 $values_i$ to $\{w\}$. By Lemma 6, all non-faulty processes decide w by round $r + 1$, which concludes the proof of the lemma. □_{Lemma 9}

Theorem 3. *The algorithm described in Figure 2 solves the binary Byzantine consensus in the system model $\mathcal{BAMP}_{n,t}[t < n/3, \diamond Synch]$.*

Proof The proof follows directly from Lemma 8 (BBC-Validity and BBC-Agreement) and Lemma 9 (BBC-Termination). \square *Theorem 3*

From asynchrony to synchrony In order to guarantee decision, after the eventual synchrony assumption holds and the timeout value at each non-faulty process is big enough (i.e., bigger than the upper bound on message transmission delay), we need that eventually all non-faulty processes execute rounds synchronously (as assumed by Lemma 9). Observe that, due to initial asynchrony, non-faulty processes can start the consensus algorithm at different instants. Moreover, due to the potential participation of Byzantine processes, some non-faulty processes can advance rounds, without deciding, while other non-faulty processes are still executing previous rounds. It is assumed that non-faulty processes may observe time at different rates and processing time is non-negligible, but is bounded by some unknown constant. By using a timeout that grows by 1 each round the following proof shows that all processes eventually reach a round from which they behave synchronously.

For the proof we will need to use a *mini-round* notation and a *catch-up mechanism*.

- **Mini-round:** Each round r is split into two mini-rounds, with the first mini-round representing lines 03 to M-05 and the second representing lines (New3) to 12. Thus, round 0 is made up of mini-rounds 0 and 1, round 1 is made up of mini-rounds 2 and 3, and so on. The reason behind splitting the rounds is so that each mini-round includes a single execution of the timer.
- **Catch-up mechanism:** A catch-up mechanism is used to help to the slow non-faulty processes to catch up to the most advanced non-faulty processes (as measured by their mini-round number).² To this end, when a process is in a mini-round ρ and receives messages corresponding to another mini-round ρ' from $(t+1)$ different processes (i.e., from at least one non-faulty process) such that $\rho' > \rho$, the process no longer waits for timers in mini-rounds $\rho, \dots, (\rho' - 1)$. It still completes these mini-rounds, but does so without waiting for timers expiration.

We assume that each process has a local clock that allows it to measure time units as integers. A process uses its local clock to measure the amount of time it waits for a timeout (where a timeout of 1 is 1 time unit). The

²Similar mechanisms are used by PBFT [16].

notation t with a subscript (for example t_{first_0}) will be used to represent a time measurement that is given by the number of time units that have passed since the algorithm started, as measured by an omniscient global observer G . By $\diamond Synchron$, processes are able to observe time at different rates, but within an unknown fixed bound. For simplicity we assume that the fastest non-faulty process observes time at a rate no faster than observed by the global observer G , thus all other processes observe time at this rate or slower. The timeouts used in the following proof are relative to the timeouts of the fastest process.

Definitions The following definitions will be used in the proofs.

- δ is a fixed, but unknown bound on message transfer delays as ensured by $\diamond Synchron$ and measured in time units as observed by G .
- t_{first_ρ} is the time, as measured by G , at which the first non-faulty process p_{first_ρ} reaches mini-round ρ (t_{first_0} is the time at which the first non-faulty process starts the consensus).
- t_{last_ρ} is the time, as measured by G , at which the last (i.e. the slowest for that mini-round) non-faulty process p_{last_ρ} reaches mini-round ρ (t_{last_0} is the time at which the last non-faulty process starts the consensus).
- θ_{fast} (resp. θ_{slow}) is the minimum (resp. maximum) amount of time, as observed by G , for any process to perform the computation of any mini-round (an unknown bounded difference between θ_{fast} and θ_{slow} is ensured by $\diamond Synchron$).
- γ_{fast_ρ} is the minimum amount of time, as observed by G , in a mini-round ρ that any process waits on line New1 or New4 before starting its timer for that mini-round.
- Mini-round ρ_δ is the first mini-round where $timeout > \delta$ at any non-faulty process.

The proof is made up of two lemmas. Lemma 10 shows that processes will eventually reach a point where they remain no more than one mini-round apart. Lemma 11 builds upon this to show that the rounds eventually become synchronous.

Lemma 10. *Consider the algorithm of Figure 2 enriched with the previous catch-up mechanism. There is a mini-round ρ_t such that in ρ_t and for all*

following mini-rounds all non-faulty processes must wait for at least part of the timeout, i.e., they do not receive $t+1$ messages from a mini-round larger than ρ_t until after they start waiting for the timeout of mini-round ρ_t .

Proof Let us consider mini-round ρ_t where $\rho_t > \rho_\delta$. For all non-faulty processes to wait at a timeout in a mini-round ρ_t , the last non-faulty process to arrive at ρ_t must arrive before it receives a message from some other non-faulty process that has already started executing a later mini-round (note that given $\rho_t > \rho_\delta$, this can only occur when the non-faulty processes are no more than 1 mini-round apart). Thus, to satisfy the lemma, a mini-round is needed where the following inequality holds at that and all following mini-rounds:

$$t_{last\rho_t} < t_{first\rho_{t+1}}. \quad (1)$$

To find out when this is satisfied first we will compute the minimum and maximum times at which non-faulty processes can arrive at a mini-round. By definition, a non-faulty process can spend no less time than $(\gamma_{fast\rho'} + \theta_{fast} + timeout_{\rho'})$ in a mini-round ρ' . Given that timeouts start with value 0 in mini-round 0 and grows by 1 in each mini-round, $timeout$ can be replaced with ρ for any mini-round ρ as a lower bound for the fastest process. We can then compute the time where the first non-faulty process arrives at mini-round ρ' (where $\rho' > \rho_\delta$) as:

$$t_{first\rho'} \geq t_{first\rho_\delta} + \left(\sum_{x=\rho_\delta}^{\rho'-1} \gamma_{fastx} + \theta_{fast} + x \right).$$

Notice that from the component $\sum_{x=\rho_\delta}^{\rho'-1} x$ (i.e., the timeout), the value of $t_{first\rho'}$ is quadratic in the number of mini-rounds.

Now consider how long it will take the slowest non-faulty process to execute mini-round ρ' when it does not wait at a timeout. By definition we know the process will spend no more time than θ_{slow} on computation. Thus, the remaining time will be spent waiting until the `wait_until()` conditions in the algorithm are satisfied. We will now examine how much time a non-faulty process can spend waiting during a mini-round on either line M-05 or M-07 (we only consider these `wait_until()` conditions as they encompass the others within a mini-round).

First consider line M-05. Its condition requires $(bin_values_i[r_i] \neq \perp)$. Given that the process is not waiting at a timeout, it must have received $(t+1)$ messages corresponding to a later mini-round, meaning that some non-faulty process has already completed ρ' . Furthermore, given that this is

the slowest non-faulty process, we know that all non-faulty processes have already executed the `BV_broadcast()` operation on line 04. As we can see in Figure 7, in the `BV_broadcast()` operation all non-faulty processes will perform at most 2 broadcast operations. Thus, by the BV-Uniformity property, all non-faulty processes will have a value in their $bin_values_i[r_i]$ after at most 2 message delays following the slowest non-faulty processes invocation of the `BV_broadcast()`. As a result, the process takes at most $2 \times \delta + \theta_{slow}$ time to execute the mini-round.

Now consider line M-07. By the time the slowest non-faulty process has reached this line all non-faulty processes have broadcast their AUX messages, thus the slowest non-faulty process will receive these AUX messages in at most δ time. The process may then need to wait for another message delay to satisfy all the conditions of line M-07 in the case where a non-faulty process had a value enter its $bin_values_i[r_i]$ immediately before broadcasting its AUX message (recall that the `BV_broadcast()` may take up to 2 message delays). Thus, as before, the process takes at most $2 \times \delta + \theta_{slow}$ time to execute the mini-round.

We then have:

$$t_{last_{\rho'}} \leq t_{last_{\rho_\delta}} + \left(\sum_{x=\rho_\delta}^{\rho'-1} 2 \times \delta + \theta_{slow} \right).$$

Notice that the value of $t_{last_{\rho'}}$ is linear in the number of mini-rounds.

Now given $t_{first_{\rho'}}$ is quadratic while $t_{last_{\rho'}}$ is linear, inequality (1) must eventually be satisfied and there will be a mini-round where all non-faulty processes wait for at least part of their timeout.

It will now be shown that for mini-rounds where $timeout > (3 \times \delta + \theta_{slow})$, once inequality (1) is true, it will remain true for all following mini-rounds. This will be done by induction. Consider $t_{last_{\rho_t}} < t_{first_{\rho_{t+1}}}$ is satisfied, let us now show that $t_{last_{\rho_{t+1}}} < t_{first_{\rho_{t+2}}}$ must also be satisfied. For this to not hold, the slowest non-faulty process must spend more time on mini-round ρ_t than the fastest non-faulty process spends on mini-round $(\rho_t + 1)$, but this is impossible because once the fastest process completes the condition on line New1 or New4 and starts its timer, $p_{last_{\rho_t}}$ must receive $(t + 1)$ messages from mini-round $(\rho_t + 1)$ after δ time. Once these messages are received, the process will not wait at any timeout, and as we have already seen, the this process will take no more than $2 \times \delta + \theta_{slow}$ time to complete the mini-round. Thus, as long as $timeout > (3 \times \delta + \theta_{slow})$, which will eventually be true given $\diamond Synch$ and the growing timeout, process $p_{last_{\rho_t}}$ will reach mini-round $(\rho_t + 1)$ before $p_{first_{\rho_{t+1}}}$ reaches mini-round $(\rho_t + 2)$. $\square_{Lemma\ 10}$

Lemma 11. *Consider the algorithm of Figure 2 enriched with the previous catch-up mechanism. Eventually the non-faulty processes attain a mini-round from which they behave synchronously.*

Proof By Lemma 10 it is known that there exists a mini-round ρ_t where at that and all following mini-rounds all non-faulty processes wait for at least part of their timeout. Additionally, this must happen at some mini-round where $timeout > (3 \times \delta + \theta_{slow})$. Consider we are in such mini-rounds. Now for a mini-round to be synchronous, all non-faulty processes need to arrive at that mini-round with enough time to broadcast their messages to all non-faulty processes before any non-faulty process moves onto the next mini-round. In the case that the last non-faulty process to arrive at the mini-round is the weak coordinator, it may take up to 3 message delays before its $COORD_VALUE[r]()$ message is received by all non-faulty processes (this includes up to 2 message delays until a value enters its $bin_values[r]$ and an additional message delay to broadcast $COORD_VALUE[r]()$). Thus, for a mini-round ρ'_t to be synchronous where $\rho'_t \geq \rho_t$, the following needs to be ensured:

$$t_{last_{\rho'_t}} + (3 \times \delta) + \theta_{slow} \leq t_{first_{\rho'_t}} + \gamma_{fast_{\rho'_t}} + timeout_{\rho'_t}. \quad (2)$$

Let us now compute $t_{last_{\rho'_t}}$. First, notice that before a non-faulty process starts its timer for a mini-round it must wait until the condition on line New1 or New4 is satisfied. Also note that by time $(t_{first_{\rho'_t}} + \gamma_{fast_{\rho'_t}} + \theta_{fast})$ at least one process has satisfied the condition on line New1 or New4 (this is given by the definition of γ). As a result all processes will receive $(t+1)$ messages from mini-round ρ'_t by time $(t_{first_{\rho'_t}} + \gamma_{fast_{\rho'_t}} + \theta_{fast} + \delta)$. Now given Lemma 10 and that $(\rho'_t - 1) > \delta$, it is known that that the slowest process is no further behind than waiting at the timeout of mini-round $(\rho'_t - 1)$. After getting these $(t+1)$ messages from mini-round ρ'_t the slow process will then skip the timeout of mini-round $(\rho'_t - 1)$ and reach the following mini-round in at most 2 additional message delays (2 message delays are needed for the same reasons given in Lemma 10 to satisfy the condition line M-05 or M-07) plus any processing time. Thus, the time at which the slowest process reaches mini-round ρ'_t is given by:

$$t_{last_{\rho'_t}} \leq t_{first_{\rho'_t}} + \gamma_{fast_{\rho'_t}} + \theta_{fast} + \theta_{slow} + (3 \times \delta).$$

Now plugging this into inequality (2) leads to $timeout_{\rho'_t} \geq (7 \times \delta) + (2 \times \theta_{slow}) + \theta_{fast}$ (note that $2 \times \theta_{slow}$ is included to account for possible processing

times in both mini-rounds $(\rho'_t - 1)$ and ρ'_t . But given that the timeout grows in each mini-round and that δ , θ_{fast} , and θ_{slow} are bound by $\diamond Synch$ there will eventually be a mini-round where this holds true.

Finally, notice that as long as the timeout is this large (i.e. $timeout \geq (7 \times \delta) + (2 \times \theta_{slow}) + \theta_{fast}$) and Lemma 10 holds then the above argument is valid for any mini-round. Now given that $timeout \geq (7 \times \delta) + (2 \times \theta_{slow}) + \theta_{fast}$ is larger than the timeout needed for Lemma 10 to hold for every following mini-round, once inequality (2), i.e. synchrony, is true for one mini-round, it will also hold for every following mini-round. $\square_{Lemma 11}$

B.4 Proof of the Blockchain Consensus (Figure 3)

Lemma 12. *There is at least one binary consensus instance that decides value 1, and all non-faulty processes exit the repeat loop.*

From an operational point of view, this lemma can be re-stated as follows: there is at least one $\ell \in [1..n]$ such that at each non-faulty process p_i , we eventually have $bin_decisions_i[\ell] = 1$.

Proof The proof is by contradiction. Let us assume that, at any non-faulty process p_i , no $bin_decisions_i[\ell]$, $1 \leq \ell \leq n$, is ever set to 1 (line 12). It follows that no non-faulty process exits the “repeat” loop (lines 02-04). As a non-faulty process p_j RB-broadcasts a valid value, it follows from the RB-Termination-1 property, that each non-faulty process p_i RB-delivers the valid proposal of p_j , and consequently we eventually have $proposals_i[j] \neq \perp$ at each non-faulty process p_i (line 11).

It follows from the first sub-predicate of line 02 and the RB-Termination-2 property that all non-faulty processes p_i invokes `bin_propose(-1)` on the BBC object `BIN_CONS[j]` and by line 11, they all BV-deliver 1 to round one. Notice that by using the RB-delivery to trigger the BV-delivery of 1 (instead of calling `BV_broadcast`) the lemma relies on the fact that the properties of `RB_broadcast` also ensure the properties of `BV_broadcast`. Namely that RB-Termination-1 ensures BV-Obligation, RB-Validity ensures BV-Justification, and RB-Termination-2 ensures BV-Uniformity and BV-Termination. It follows that the properties of the binary consensus are maintained. Hence, from its BBC-Termination, BBC-Agreement, BBC-Validity, and Intrusion-tolerance properties (as no non-faulty process has proposed 0), this BBC instance returns the value 1 to all non-faulty processes, which exit the “repeat” loop.

$\square_{Lemma 12}$

Lemma 13. *A decided value is a valid value (i.e., it satisfies the predicate $\text{valid}()$).*

Proof Let us first observe that, for a value $\text{proposals}_i[j]$ to be decided by a process p_i , we need to have $\text{bin_decisions}_i[j] = 1$ (lines 08-10).

If the value 1 is decided by $\text{BIN_CONS}[j]$, $\text{bin_decisions}_i[j] = 1$ is eventually true at each non-faulty process p_i (line 12). It follows from (i) the fact that the value 1 can only enter the bin_values of a BBC instance after validation at line 11, and (ii) the Intrusion-tolerance property of $\text{BIN_CONS}[j]$, that at least one non-faulty process p_i inserted 1 into its binvalues on line 12. Due to line 11, it follows that $\text{proposals}_i[j]$ contains a valid value. $\square_{\text{Lemma 13}}$

Lemma 14. *No two non-faulty processes decide different values.*

Proof Let us consider any two non-faulty processes p_i and p_j , such that p_i decides $\text{proposals}_i[k1]$ and p_j decides $\text{proposals}_j[k2]$. It follows from line 08 that $k1 = \min\{x \text{ such that } \text{bin_decisions}_i[x] = 1\}$ and $k2 = \min\{x \text{ such that } \text{bin_decisions}_j[x] = 1\}$.

On the one hand, it follows from line 07 that $(\bigwedge_{1 \leq x \leq n} \text{bin_decisions}_i[x] \neq \perp)$ and $(\bigwedge_{1 \leq x \leq n} \text{bin_decisions}_j[x] \neq \perp)$, from which we conclude that both p_i and p_j know the binary value decided by each binary consensus instance (line 12). Due to the BBC-Agreement property of each binary consensus instance, we also have $\forall x : \text{bin_decisions}_i[x] = \text{bin_decisions}_j[x]$. Let $\text{dec}[x] = \text{bin_decisions}_i[x] = \text{bin_decisions}_j[x]$. It follows then from line 08 that $k1 = k2 = \min\{x \text{ such that } \text{dec}[x] = 1\} = k$. Hence, $\text{dec}[k] = 1$.

On the other hand, it follows from the Intrusion-tolerance property of $\text{BIN_CONS}[k]$ that a non-faulty process p_ℓ inserted 1 into its binvalues on line 12. As this invocation can be issued only at line 03, we conclude (from the predicate of line 02) that $\text{proposals}_\ell[k] = v \neq \perp$. As p_ℓ is non-faulty, it follows from the RB-Unicity and RB-Termination-2 properties that all non-faulty processes RB-delivers v from p_k . Hence, we eventually have $\text{proposals}_i[k] = \text{proposals}_j[k]$, which concludes the proof of the lemma.

$\square_{\text{Lemma 14}}$

Lemma 15. *Every non-faulty process decides a value.*

Proof It follows from Lemma 12 that there is some p_j such that we eventually have $\text{bin_decisions}_i[j] = 1$ at all non-faulty processes, and no non-faulty process blocks forever at line 04. Hence, all non-faulty processes invoke each binary consensus instance (at line 03 or line 06). Moreover, due to

their BBC-Termination property, each of the n binary consensus instances returns a result at each non-faulty process (line 12). It follows that no non-faulty process p_i blocks forever at line 07. Finally, as seen in the proof of Lemma 14, the predicate of line 09 is eventually satisfied at each non-faulty process, which concludes the proof of the lemma. $\square_{\text{Lemma 15}}$

Theorem 4. *The algorithm described in Figure 3 implements multivalued Byzantine consensus (VPBC) in the system model $\mathcal{BAMP}_{n,t}[t < n/3, \text{BBC}]$.*

Proof Follows from Lemma 13 (VPBC-Validity), Lemma 14 (VPBC-Agreement), and Lemma 15 (VPBC-Termination). $\square_{\text{Theorem 4}}$

B.5 Complexity

The proposed reduction has constant time complexity.

Lemma 16. *When `RB_broadcast` is invoked by a non-faulty process, all non-faulty processes `RB_deliver` the value in constant time.*

Proof Within `RB_broadcast`, a process starts by calling `broadcast` with a value, which will then be echoed by all non-faulty processes, resulting in all processes delivering the value in 3 communication steps. $\square_{\text{Lemma 16}}$

Lemma 17. *For any process that `RB_delivers` a value, within a constant amount of time following this all non-faulty processes have `RB_delivered` the value.*

Proof For a process to `RB_deliver` a value it must have received the value from $n - t$ processes, thus $t + 1$ non-faulty processes have `broadcast` this value, which means all non-faulty will echo that value and all non-faulty will receive the value from $n - t$ processes in at most 2 communication steps following the first `RB_delivery`. $\square_{\text{Lemma 17}}$

Theorem 5. *The reduction presented in figure 3 is a constant time reduction.*

Proof Let us show this by contradiction. Assume a non-faulty process does not decide in constant time, there are two possibilities how this could happen, either: (i) the process waits on line 09 for more than constant time, or (ii) the process has not invoked some instance `BIN_CONS[k]` until after constant time had already passed.

First consider (i). Here the process is waiting to `RB_deliver` a value that by lemma 13 has already been `RB_deliverd` by some non-faulty process, but by lemma 17 we know this must happen in constant time.

Now consider (ii). By lemma 12 we know that at least one binary consensus instance decides 1. Once this happens all non-faulty processes invoke all remaining instances of `BIN_CONS`[k] without waiting. Thus, for (ii) to be true, no instance of `BIN_CONS`[k] must have terminated with 1 in constant time. But given that at least $2t + 1$ instances of `RB_broadcast` will be invoked by non-faulty processes, and given lemma 16, all non-faulty processes will invoke all instances of `BIN_CONS`[k] in constant time on line 03 or 06.

□*Theorem 5*

C The BV-broadcast all-to-all communication implementation

Figure 7 depicts the pseudocode of an existing implementation [38] of the BV-broadcast problem stated in Section 3.1.

```

operation BV_broadcast MSG( $v_i$ ) is
(01) broadcast B_VAL( $v_i$ ).

when B_VAL( $v$ ) is received
(02) if (B_VAL( $v$ ) received from  $(t + 1)$  different processes and B_VAL( $v$ ) not yet broadcast)
(03)   then broadcast B_VAL( $v$ ) // a process echoes a value only once
(04) if (B_VAL( $v$ ) received from  $(2t + 1)$  different processes)
(05)   then  $bin\_values_i \leftarrow bin\_values_i \cup \{v\}$ . // local delivery of a value

```

Figure 7: An algorithm implementing BV-broadcast in $\mathcal{BAMP}_{n,t}[t < n/3]$ (from [37])

D Reliable broadcast in Byzantine systems

This broadcast abstraction (in short, RB-broadcast) was proposed by G. Bracha [10]. It is a one-shot one-to-all communication abstraction, which provides processes with two operations denoted `RB_broadcast()` and `RB_deliver()`. When p_i invokes the operation `RB_broadcast()` (resp., `RB_deliver()`), we say that it “RB-broadcasts” a message (resp., “RB-delivers” a message). An RB-broadcast instance, where process p_x is the sender, is defined by the following properties.

- RB-Validity. If a non-faulty process RB-delivers a message m from a non-faulty process p_x , then p_x RB-broadcast m .
- RB-Unicity. A non-faulty process RB-delivers at most one message from p_x .
- RB-Termination-1. If p_x is non-faulty and RB-broadcasts a message m , all the non-faulty processes eventually RB-deliver m from p_x .
- RB-Termination-2. If a non-faulty process RB-delivers a message m from p_x (possibly faulty) then all the non-faulty processes eventually RB-deliver the same message m from p_x .

The RB-Validity property relates the output to the input, while RB-Unicity states that there is no message duplication. The termination properties state the cases where processes have to RB-deliver messages. The second of them is what makes the broadcast reliable. It is shown in [11] that $t < n/3$ is an upper bound on t when one has to implement such an abstraction.

Let us remark that it is possible that a value may be RB-delivered by the non-faulty process while its sender is actually Byzantine and has not invoked `RB_broadcast()`. This may occur for example when the Byzantine sender played at the network level, at which it sent several messages to different subsets of processes, and the RB-delivery predicate of the algorithm implementing the RB-broadcast abstraction is eventually satisfied for one of these messages. When this occurs, by abuse of language, we say that the sender invoked RB-broadcast. This is motivated by the fact that, in this case, a non-faulty process cannot distinguish if the sender is faulty or not.

The algorithm described in [10] implements RB-broadcast in $\mathcal{BAMP}_{n,t}[t < n/3]$. Hence, it is t -resilience optimal. This algorithm requires three communication steps to broadcast an application message.